Lecture 11 11/5/20 ①

Left Linear Grammar from FA

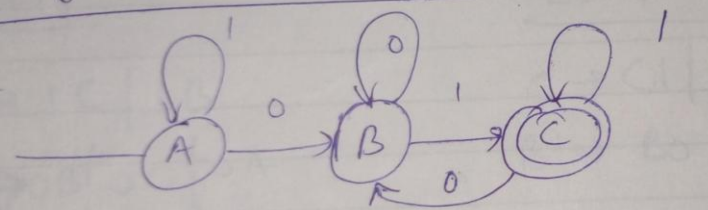
Steps

① obtain the reverse of given DFA

② obtain the right linear grammar from the reversed DFA.

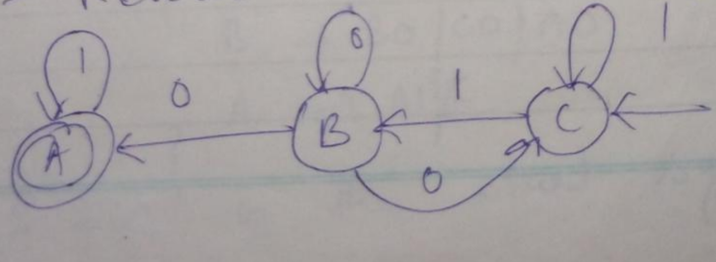
③ obtain left linear grammar from RLG.

Ex.



Step 1 Reverse the DFA, make A as the final state and C as the start state

- Reverse the direction of arrows.



Step 2. obtain RLG from the reversed DFA. (2)

$C \to 1C \mid 1B$

$B \to 0B \mid 0C \mid 0A$

$A \to 1A \mid \epsilon$

Step 3 reverse the productions.

RLG	LLG
$C \to 1C \mid 1B$	$C \to C1 \mid B1$
$B \to 0B \mid 0C \mid 0A$	$B \to B0 \mid C0 \mid A0$
$A \to 1A \mid \epsilon$	$A \to A1 \mid \epsilon$

$V = \{C, A, B\}$

$T = \{0, 1\}$

$P = \{ C \to C1 \mid B1$
$B \to B0 \mid C0 \mid A0$
$A \to A1 \mid \epsilon \}$

$S = C$ is the start symbol.

What ever string is accepted by original DFA is accepted by the reversed DFA.

Proof :→ Consider 10101

$C \to B1$	$C \to B1$
$\to C01$	$B \to C0$
$\to B101$	$C \to B1$
$\to A0101$	$B \to A0$
$\to A10101$	$A \to A1$
$\to 10101$	$A \to \varepsilon$

B1

C01

B101

A0101

A10101

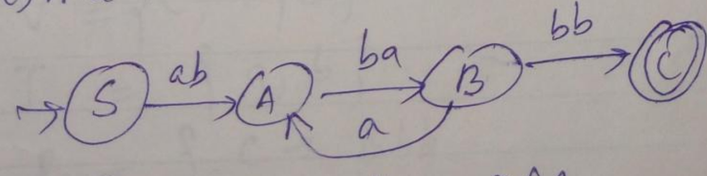
Ex: Obtain a LLG for the RLG

$S \to abA$

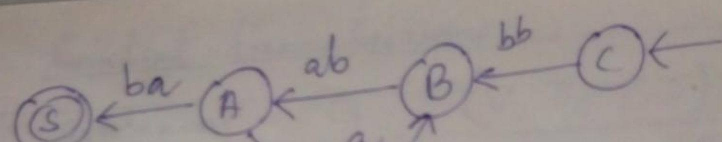
$A \to baB$

$B \to aA \mid bb$.

Sol Construct the DFA



Step 1 Reverse the DFA



step 2 obtain the RLG. for reversed DFA.

$C \rightarrow bbB$

$B \rightarrow abA$

$A \rightarrow ba \mid aB$

step 3 . Reverse the productions

RLG

$C \rightarrow bbB$

$B \rightarrow abA$

$A \rightarrow ba \mid aB$

LLG

$C \rightarrow Bbb$

$B \rightarrow Aba$

$A \rightarrow ab \mid Ba$

$V = \{C, A, B\}$

$T = \{a, b\}$

$P = \{C \rightarrow Bbb$
$B \rightarrow Aba$
$A \rightarrow ab \mid Ba\}$

$S = C$ is the start symbol

Context free Grammar

(a) A grammar $G = (V, T, P, S)$ is said to be type 2 or CFG if all the productions are of the form

$$A \rightarrow \alpha$$

$\alpha \in (V \cup T)^*$ and $A \in V$, and

ϵ can appear on the right hand side of any production.

Observations

(1) There is only one symbol A on the LHS of the production and that symbol must be a non-terminal

(2) $\alpha \in (V \cup T)^*$ implies that RHS of the string may contain any number of terminal and non-terminals including ϵ (null string)

Ex

$$S \rightarrow aBaa \mid bA \mid \epsilon$$

$$A \rightarrow aA \mid bAA$$

$$B \rightarrow BbB \mid a \mid \epsilon$$

* Every regular grammar is a CFG and hence a regular language is also CFL, but reverse is not true.
* $\therefore$ RG is a subset of CFG and RL is a subset of CFL.

Ex $G = (V, T, P, S)$

$V = \{S\}$

$T = \{a, b\}$

$P = \{ S \to aSa \mid bSb \mid \epsilon \}$

S is the start symbol.

What is the language generated by ~~The null str~~ this grammar?

Sol: null string ϵ can be obtained by ~~$S \to \epsilon$~~ ~~can be obtained by~~ applying the production $S \to \epsilon$

$S \to \epsilon$

$S \to aSa$

$\to abSba$

$\Rightarrow abbSbba$

$\Rightarrow abbbSbbba \quad - \quad abbbbba$

* So by applying the production
$S \to aSa$ and $S \to bSb$
any number of times and in any order and finally applying the production $S \to \epsilon$.
we get a string w followed by reverse of w denoted by w^R

$$L = \{ ww^R \mid w \in (a+b)^* \}$$

As the language is generated from the CFG, this is context free language.

Q) Show that the language
$L = \{ a^m b^n \mid m \neq n \}$ is context free

Sol. If it is possible to construct a CFG to generate the given language then the language is context free.

$S \to aSb$ and
$S \to \epsilon$ ⇒ m # of a's followed by S and n # of b's

But $m \neq n \Rightarrow$ an extra a or extra b

$S \rightarrow \varepsilon$

$S \rightarrow A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b.$

$G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b\}$

$P = \{ S \rightarrow aSb \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$\}$

S is the start symbol

$L = \{ a^m b^n \mid m \neq n \}$

As a CFG exists $\Rightarrow$ lang is CF.

Obtain a CFG to generate unequal number of a's and b's. (9)

Sol.

$$L = \{ w \mid w \in \{a,b\}^* \; n_a(w) \neq n_b(w) \}$$

G_1 — more a's

G_2 — more b's

combine.

$S \to A \mid B$

$\underbrace{A \to a \mid aA}$ only a's not b's

include b's

$A \to bAA \mid AbA \mid AAb$

$\underbrace{B \to b \mid bB}$. only b's not a's

include. a's

$B \to aBB \mid BaB \mid BBa$

$\therefore$

$A \to a \mid aA \mid bAA \mid AbA \mid AAb$

$B \to b \mid bB \mid aBB \mid BaB \mid BBa$.

$G = G_1 G_2$

$G = (V, T, P, S)$

$T = \{a, b\}$.

$$P = \{ S \to A \mid B, \; A \to a \mid aA \mid bAA \mid AbA \mid AAb, \; B \to b \mid bB \mid aBB \mid BaB \mid BBa \}$$

$S \to$ start

obtain CFG for a RE $(011+1)^*(01)^*$ (10)

Sol.

$(011+1)^* \; (01)^*$

$A^* \quad B^*$

$A = 011$ or 1

$B = 01$

$A \rightarrow 011A \mid 1A \mid \varepsilon$

$B \rightarrow 01B \mid \varepsilon$

$S \rightarrow AB$

$G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{0, 1\}$

$P = \{$

$S \rightarrow AB$

$A \rightarrow 011A \mid 1A \mid \varepsilon$

$B \rightarrow 01B \mid \varepsilon$

$\}$

S - start symbol